

# TURBOMATHS SUPPORT CLASSES

University of KwaZulu-Natal

## MATHEMATICS Grade 10 Lecture 4 – 6

### FACTORS AND PRODUCTS

#### PRODUCTS

The instructor will consider a number of examples on the following types of products. Then you will be required to complete the exercise below.

- A. Monomial  $\times$  binomial
- B. Binomial  $\times$  binomial
- C. Product of Sum and Difference
- D. Squaring the binomial
- E. Product of polynomial by binomial

#### EXERCISE

Multiply

1.  $(2x + 3y)(2x - 5y)$
2.  $2x^2 + 4x + 16$  by  $3x - 4$
3.  $3y^2 - 3y + 9$  and  $2y + 3$
4.  $4a^2b - 2ab + 3b^2$  and  $ab - 2b$
5.  $\left(4x^2 - \frac{1}{2}y\right)\left(3x + \frac{1}{4}y\right)$
6.  $(2x + 3y)^2$
7.  $(2x^3 + 3y^2)^2$
8.  $(4x^2 - 5y)^2$
9.  $(2x^2 + 5xy)(2x^2 + 5xy)$
10.  $(2x + y)(2x - y)(4x^2 + y^2)$

#### FACTORISATION

#### COMMON FACTORS

Examples by instructor

#### EXERCISE 1

Factorise by taking out a common factor, and then simplify the factors if possible:

- 1.1  $(2x + 1)(x + 3) - (x + 3)$
- 1.2  $(2x - 3)^2 - (2x - 3)$
- 1.3  $(x - 5)^3 - (x - 5)^2$

- 1.4  $(x - y)^2 - (x - y)$
- 1.5  $3x^2(x - y) - ax^3(x - y)$
- 1.6  $3(x - y)^2 - (x - y)$
- 1.7  $2(a - b)^2 - (2a - b)(a - b)$
- 1.8  $2x(3x - 1)^3 - (3x - 2)(3x - 2)(3x - 1)^2$

#### DIFFERENCE OF TWO SQUARES

Examples by instructor

#### EXERCISE 2

Factorise completely :

- 1.9  $9x^2 - 16y^2$
- 1.10  $(2x - y)^2 - (x - y)^2$
- 1.11  $4x^6y^2 - 1$
- 1.12  $x^{16} - y^2$
- 1.13  $3x^{16} - 27$
- 1.14  $2x^8 - 8y^4$
- 1.15  $2x^2 - \frac{1}{2}$
- 1.16  $(x + 1)^2 - (x - 1)^2$
- 1.17  $x^{2n} - 1$

#### MORE CHALLENGING

#### DIFFERENCE OF TWO SQUARES

#### EXERCISE 3

Factorise completely:

- 1.18  $(4x - 3)^2 - (2x - 1)^2$
- 1.19  $(3x - 2)^2 - (x - 6)^2$
- 1.20  $(4x - 1)^2 - (2x - 3)^2$
- 1.21  $(3x - 5)^2 - (6x - 4)^2$
- 1.22  $(4a - 3b)^2 - (5a + 6b)^2$
- 1.23  $4(a - b)^2 - 36(a + b)^2$
- 1.24  $(5a + b - c)^2 - (3a - b + c)^2$
- 1.25  $8x^2 - 18(x - 2y)^2$
- 1.26  $3x^8 - 3$
- 1.27  $16x - 9x^3$
- 28.1  $(x + y)(x^2 - xy + y^2)$
- 28.2  $(x - y)(x^2 + xy + y^2)$

## SQUARING A BINOMIAL

### EXERCISE 4

Expand:

- 1.28  $(x + 3)^2$   
1.29  $(3a - 2)^2$   
1.30  $(2x - 3y)^2$   
1.31  $(a - 1)^2$   
1.32  $(5x - 2)^2$

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## FACTORS BY GROUPING

### EXERCISE 5

Factorise:

- $mx + 2my - 3x - 6y$
- $x^3 - x^2y - 3x + 3y$
- $(x - 5y)^2 - x^2 + 5xy$
- $(x - y)^2 - x + y$
- $a^2 - b^2 + a + b$
- $a^2 - b^2 - a + b$
- $a^2 - b^2 - a + b$
- $a^2 - b^2 - a - b$
- $(x - y)^2 + x^2 - y^2$
- $(x - y)^2 - x^2 + y^2$
- $3ab + 2 - a - 6b$
- $x^2 - x - y^2 - y$

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## MORE CHALLENGING FACTORS BY GROUPING

### EXERCISE 6

Factorise:

- $6a^2 + 2bc + 3ac + 4ab$
- $3x - 2ym - mx + 6y$
- $6a^2 - 9ab - 4a + 6b$
- $6x^2 + 3y - 9x - 2xy$
- $5x^2 - 9y + 3x - 15xy$
- $6x - 9y^2 + 6xy - 9y$
- $x^2 - a - 1 - ax$
- $6ab - 2a - 4b + 3a^2$
- $6x^2 + y - 3x - 2xy$

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## MORE EVIL FACTORS BY GROUPING

### EXERCISE 7

Factorise: (only some of these will need switch-rounds)

- $ax - ay - by + bx$
- $px - py + y - x$
- $a^2 - b^2 - b + a$
- $x^2 - ax - 1 + a$
- $x^2 + 9ax - 4 - 18a$
- $x^2 + 3x + bx - 3b$
- $x^3 - x^2y - y^3 + xy^2$
- $(x^2 - 3)(x^2 - x - 6) - (x + 3)(x^2 - x - 6)$
- $x(x - 1) - y$
- $6a^2 + 6b - 4a - 9ab$
- $6x^2 + 3y - 9x - 2xy$
- $5x^2 - 9y + 3x - 15xy$
- $6x - 9y^2 + 6xy - 9y$
- $x(x - 1) - y(y - 1)$
- $x^2y - x^2 - xy^2 + xy$
- $x^3 - 3a^2b + 3ab^2 - b^3$
- $x(x^2 - 1) - y(y^2 - 1)$
- $2x^3 + 3x^2 - 6x - 16$
- $a^2 - 2ab + 2bc - c^2$
- $1 - 3x - 3xy - y^2$

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## THE QUADRATIC TRINOMIAL

Examples

### EXERCISE 8

Factorise:

- $15x^2 - 31x + 14$
- $2x^2 + 7x + 6$
- $6x^2 - 13x + 6$
- $6x^2 - 19x + 3$
- $9x^2 + 18x + 8$
- $12x^2 - 35x + 8$
- $8x^2 + 73x + 9$
- $18x^2 - 39x + 6$

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## MORE QUADRATIC TRINOMIALS

### EXERCISE 9

- $5x^2 + x - 6$
- $3x^2 - 5x - 2$
- $6x^2 - 5x - 6$
- $6x^2 + 5x - 6$

5.  $18x^2 - 9x - 2$
6.  $18x^2 + 5x - 2$
7.  $12x^2 - x - 6$
8.  $8x^2 - 47x - 6$

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### **YET MORE QUADRATIC TRINOMIALS**

#### **EXERCISE 10**

1.  $9x^2 - 3x - 2$
2.  $9x^2 - 3x - 20$
3.  $12x^2 + 11x + 2$
4.  $6x^2 - 13x + 6$
5.  $5x^2 + 21x + 18$
6.  $12x^2 - 11x + 2$
7.  $18x^2 - 39x + 20$
8.  $18x^2 - 53x + 20$
9.  $24x^2 - 41x + 12$
10.  $2x^2 - 4x + 15$
11.  $24x^2 + 2x - 15$
12.  $8x^2 - 31x - 4$
13.  $8x^2 - 22x + 9$
14.  $8x^2 + 3x + 6$
15.  $24x^2 + 7x - 6$
16.  $8x^2 - 22x + 9$
17.  $12x^2 - 56x + 9$
18.  $6x^2 - 37x + 6$
19.  $6x^2 - 29x - 5$
20.  $6x^2 - 47x - 9$
21.  $12x^2 - 8x - 15$
22.  $36x^2 - 45x - 4$
23.  $24x^2 + 35x + 4$
24.  $48x^2 - 55x - 12$
25.  $60x^2 - 19x - 48$

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### **HARDER TRINOMIALS**

#### **EXERCISE 11**

Factorise:

1.  $8x^2 - 10xy + 3y^2$
2.  $18x^2 - 5xy - 2y^2$
3.  $18x^2 + 12xy - y^2$
4.  $12 - 26x - 10x^2$
5.  $8 - 26x + 15x^2$
6.  $8a^2 + 13ab - 6b^2$

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### **SINISTER TRINOMIALS**

### **QUESTION 12**

Factorise:

1.  $(a - b)^2 - 5(a - b) + 6$
2.  $2(a - b)^2 - 5(a - b) - 3$
3.  $(2a - b)^2 - 3(2a - b) - 4$
4.  $(x - 3y)^2 + 8(x - 3y) + 15$
5.  $3(m + 2f)^2 + 5(m + 2f) - 2$
6.  $6(2x - 1)^2 - 11(2x - 1) + 3$
7.  $6(x - 2)^2 + 5(x - 2) - 6$
8.  $8(3a - 1)^2 - 10(3a - 1) + 3$
9.  $18(2x - 3)^2 - 5(2x - 3) - 2$
10.  $18(2x - 5)^2 + 21(2x - 5) - 4$
11.  $8 - 26(x - 2) + 15(x - 2)^2$
12.  $8x^2 + 13xy - 6y^2$
13.  $12 - 26x - 10x^2$
14.  $6x^4 + 47x^2 - 8$

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### **EVIL TRINOMIALS**

#### **QUESTION 13**

Factorize completely:

1.  $(x^2 - 2x)^2 - 2(x^2 - 2x) - 3$
2.  $(a^2 + 4a)^2 - 2(a^2 + 4a) - 24$
3.  $(3x^2 - 2x)^2 - 9(3x^2 - 2x) + 8$
4.  $(x^2 - 5x)^2 - 8(x^2 - 5x) - 84$
5.  $(2x^2 - 3x)^2 - 11(2x - 3x) + 18$
6.  $2(2a + b)^2 + (2a + b)(a + 2b) - (a + 2b)^2$
7.  $24 - 11xy - 18x^2y^2$
8.  $60 + 53y^2 - 36y^4$
9.  $12 + 8(2x + 3) - 15(2x + 3)^2$
10.  $40 + 28(3x - 2x^2) + 4(3x - 2x^2)^2$

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### **MIXED FACTORISATION**

#### **QUESTION 14**

1.  $12x^2 - 19x - 18$
2.  $8x^3 - 18x$
3.  $3 - 27x^4$
4.  $(3x - 2)^2 - (2x + 1)^2$
5.  $(a - b)^3 - 2(a - b)^2 - 3(a - b)$
6.  $x^3 + 2x^2 - x - 2$
7.  $6a^2 - 9ab - 4a + 6b$
8.  $5x^2 - 9y + 3x - 15xy$

9.  $x^2y - x^2 - xy^2 + xy$
10.  $(a^2 - b^2)^2 - (a - b)^2$
11.  $x^3 - x^2 - y^3 + y^2$
12.  $x^3 + (3x - 1)^3$
13.  $8x^3 - 12x^2 + 6x - 1$
14.  $9(2x - y)^2 - (x - y)^2$
15.  $2a^2 - 6bc + 3ab - 4ac$

## FIENDISH GROUPINGS

### QUESTION 15

1.  $x^2 - 2x + 1 - y^2$
2.  $a^2 - 4ab + 4b^2 - c^2$
3.  $x^2 - a^2 + 2ab - b^2$
4.  $9y^2 - 1 + 4x - 4x^2$
5.  $x^2 - a^2 + 9y^2 - 6xy$
6.  $9 - a^2 + 4x^2 - 12x$

## CUMULATIVE REVIEW

### QUESTION 16

Factorise:

1.  $2x^2 - x - 10$
2.  $10x^2 - 19x - 15$
3.  $(2x - 5)^2 - 1$
4.  $5x^2 - 15x - 20$
5.  $x^2 - y^2 - mx - my$
6.  $3a^3b - 2a^2b^2 - ab^3$
7.  $x^2 - y^2 + x + y$
8.  $x^2 - a + ax - x$
9.  $2ax + 3bx - 4ay - 6by$
10.  $ac - ab - bc + c^2$
11.  $6ab - 9ac + 6cd - 4bd$
12.  $6x^2 + 3xy - 9x - 2x^2y$
13.  $x^2y - x^2 - xy^2 + xy$
14.  $12ax^2 + 8ax - 15a$

## MORE REVIEWS

### QUESTION 17

1.  $6ab - 2a - 4b + 3a^2$
2.  $x^2 + y - x - y^2$
3.  $x^2 + y - x - y^2$
4.  $x^2 + ax - 3a - x - 6$
5.  $16x^3 - 54y^3$
6.  $3a + 6 + 2x^2 - 7x - 2ax$
7.  $3xy^2 - 6xy + 9x^2 - 2y^3$

8.  $(x^2 - y^2)^2 - (x^2 - y^2)$
9.  $x^2(x + 1) + y^2(y + 1) + 2xy$
10.  $x - y^2 - x^2 + 2xy - y$
11.  $x - y^2 - x^2 + 2xy - y$
12.  $a^3 - ab + b^2 - ab^2$

## SIMPLIFICATION (MONOMIAL DENOMINATORS)

### QUESTION 18

- |   |                                 |
|---|---------------------------------|
| 1. $\frac{x^5}{x^2}$                    | 2. $\frac{x^6}{x^3}$            |
| 3. $\frac{12x^5}{4x^2}$                 | 4. $\frac{8x^5}{4x^4}$          |
| 5. $\frac{x^5}{x}$                      | 6. $\frac{x^{n+4}}{x^2}$        |
| 7. $\frac{x^{n+1}}{x}$                  | 8. $\frac{x^{n+1}}{x^n}$        |
| 9. $\frac{pq}{p}$                       | 10. $\frac{5+4}{5}$             |
| 11. $\frac{5+x}{5}$                     | 12. $\frac{p+q}{q}$             |
| 13. $\frac{x^{n+7}}{x^9}$               | 14. $\frac{x^{8q}}{x^9}$        |
| 15. $\frac{x^3y^4}{xy}$                 | 16. $\frac{12x^5y^8}{3x^3y^2}$  |
| 17. $\frac{xa + xb}{a + b}$             | 18. $\frac{ax + b}{x + b}$      |
| 19. $\frac{x^2 - 2xy + y^2}{x^2 - y^2}$ | 20. $\frac{2x^2 - x}{8x^2 - 2}$ |

## SIMPLIFICATION OF ALGEBRAIC FRACTIONS

### EXERCISE 19

1.  $\frac{x-3}{3} + \frac{3x-1}{12}$
2.  $\frac{x-2}{2} - \frac{x+1}{6} - \frac{x-3}{3}$
3.  $\frac{a+b}{5} - \frac{a-b}{4}$
4.  $4 + \frac{x-3}{2}$
5.  $1 - \frac{2x-5}{3}$
6.  $\frac{1}{6}(5a+3) + \frac{1}{3}(2a-3) - \frac{1}{5}(3a-5) - \frac{1}{10}(7a+5)$
7.  $\frac{5}{6}(x-1) - \frac{x-1}{2} + \frac{x-3}{3}$