

TURBOMATHS SUPPORT CLASSES

University of KwaZulu-Natal

MATHEMATICS Grade 10

MATHEMATICS OF FINANCE

SIMPLE INTEREST

Interest is the money you pay someone for the use of their money. There are 2 ways that interest is calculated. Simple interest or compound interest.

Simple interest is based on the entire amount of the loan over the total period of the loan. The simple interest formula is

$$\text{Simple Interest} = P \times R \times T$$

where P = the principal (that is the amount being borrowed)

R = the rate (in percent \div 100) at which the money must be repaid.

T = the time period over which the loan has to be repaid.

Another way of expressing this is as follows. Suppose a certain sum of money (P) is invested at i % interest over a period of n years. Then the total amount one will get after n years is called the accumulated amount (A) and the amount is obtained using the formula:

$$A = P(1 + ni)$$

Example 1: Finding the Interest payable

Sizwe borrows R1200 from a lender at 24% interest per annum. He will repay the loan and interest after 9 months. How much interest does he pay? What is the total amount he pays.

Example 2:

You invest R10 000 at a simple interest rate of 12% per annum for 8 months. How much does the investment grow to?

Example 3: Finding the rate

Patrick lent Alice R300. After 6 months she paid him back R330. What annual rate of interest did Patrick charge?

TURBO-FACT: HAVE YOU EVER WONDERED ...?

A raw deal has often been described as getting the "short end of the stick". Have you ever wondered why?

TURBO-FACT: Until 1826 British banks used a tally stick to keep accounts. The stick had notches and was then split into two pieces. The lender kept the longer piece and the borrower the shorter piece. At certain times

they would put both pieces together to check transactions. Why not check out the details by doing some research on "tally sticks".

EXERCISE

- Find the simple interest in each case below:
 - $P = R420; R = 9\%$ p.a, $T = 3$ years
 - $P = R520, R = 6,5\%$ p.a, $T = 4$ years
 - $P = R587, R = 0,045\%$ per day, $T = 30$ days
 - $P = R12752, R = 1,5\%$ per month, $T = 9$ months
- R1300 is invested at $r\%$ interest p.a over two years and accumulates interest of R104. Find r .
- How much must be invested at 6% interest p.a over 3 months to achieve an interest of R12 000.
- Over what period must R800 be invested at 6% p.a to produce interest of R64?
- In what time will a sum of R100 at 12% yield an Interest of R48.
- Simple Interest of R60, Principal of R500 and for 2 years, Find the rate of interest.
- In what time will a sum of R600 at 13% yield an Interest of R78.
- Simple Interest of R100, Principal of R500 and for 4 years, Find the rate of interest.
- Simple Interest of R540 at 9% and for 2 years, Find the principal.
- Simple Interest of R72, Principal of R300 and for 4 years, Find the rate of interest.
- Simple Interest of R15 at 3% and for 1 year, Find the principal.
- Simple Interest of R800, Principal of R2000 and for 4 years, Find the rate of interest.

COMPOUND INCREASE

Meaning of Compound Increase:

- (1) Compound increase formula:

$$A = P \left(1 + \frac{r}{100} \right)^n$$

A is the accumulated amount; that is the amount you get at the end of the investment period.

P is the principal amount; that is the amount you invest at the beginning.

r is the rate per period (%).

eg. for 'annually' we divide the % by 1, for

'monthly' we divide the % by 12, for 'quarterly' we divide the % by 4.

n is the number of time periods

eg. For 3 years compounded monthly we use n

$$= 3 \times 12 = 36.$$

- (2) To calculate the rate of interest r we adjust the above formula:

$$r = 100 \left(\sqrt[n]{\frac{A}{P}} - 1 \right).$$

Example 1: Mr. Ndlovu invests R10 000 in a bank which offers him an interest rate of 10% per annum compounded monthly. He thinks that he will need R16 000 for his son's university fees in 5 years time. Will his investment be enough?

SOLUTION: We must calculate the value of his investment after 5 years. We use formula (1) with $P =$

$$R10\,000; r = \frac{10}{12}$$

(Monthly); $n = 5 \times 12 = 60$ intervals.

$$\begin{aligned} \text{We thus get } A &= P \left(1 + \frac{r}{100} \right)^n \\ &= 10000 \left(1 + \frac{10}{1200} \right)^{60} \\ &= R16453,09 \end{aligned}$$

From this it is clear that he will have enough for his need. In fact he will have more than R400 extra.

USING THE CALCULATOR

Can you use your calculator well? How did we get the answer from the second last step?

Follow these steps:

- (1) Type $10 \div 1200$ and press =.
The screen shows 0,0083333
- (2) Now press + 1. The screen shows 1,0083333
- (3) Thereafter press the exponent key (y^x) and then 60.
The screen shows 1,6453089
- (4) Now press $\times 10\,000$.
The screen shows 16453.089.
- (5) Round this off to the nearest cent to give R16 453,09.

Example 2: R15 000 is invested at an interest rate of r % per year. The interest is compounded yearly and the investment will double in 4 years. Calculate the value of r correct to one decimal digit.

SOLUTION: Here we have $P = R15\,000$; The interest rate is r % and interest is compounded yearly so the rate is just r . We are also told that the investment will double in 4 years. So we know $A = R30\,000$ and $n = 4$. We use formula 2:

$$\begin{aligned} r &= 100 \left(\sqrt[4]{\frac{A}{P}} - 1 \right) \\ &= 100 \left(\sqrt[4]{\frac{30000}{15000}} - 1 \right) \\ &= 18,92\% \end{aligned}$$

To 1 decimal place the answer is 18,9%.

USING THE CALCULATOR

To obtain the answer quickly with a calculator, follow these steps:

1. Type $30\,000 \div 15\,000$.
The screen shows 2.
2. Now press the root key ($\sqrt[x]{\quad}$) and then 4. (You may have to press 2ndF first). The screen shows 1,1892071
3. Next press - 1. The screen shows: 0,1892 ...
4. Now $\times 100$. You get 18,92.....

PRACTICE PROBLEMS

1. Tledima invested R 5 000 for his son Tokelo when he was three years old. If interest is compounded annually at the rate of 8% per annum, determine how old Tokelo will be when his investment amounts to four times the original amount. (5)
2. Hope inherited a sum of money which he invested at 11,5% per annum calculated monthly. This was a fixed investment for 12 years and at the end of the 12th year he had R12 528 in the bank. How much did he invest, to the nearest rand? (5)
3. Vuwani buys a bicycle which costs R2 000. He tells the seller that he will pay the sum over a period of one year. The seller calculates what he would get if he invested R2 000 for one year at an interest of 24% compounded monthly, and Vuwani agrees to pay this in twelve equal instalments.
- 3.1 Determine how much Vuwani has to pay altogether (rounded off to the nearest cent). (6)
- 3.2 How much does Vuwani pay per month? (1)
4. A man invests R10 000 for a period of 3 years. He is offered a rate of 10% per annum compounded quarterly or 10,5% per annum compounded annually.
 - 4.1 What is the final amount on each investment? (5)
 - 4.2 Which is the better investment? (1)

EXERCISE SET 1

1. Find the compound interest and amount for a principal of R200 at 6% per year, for 3 years
2. Compound Interest of R75.31 at 3% per year, for 4 years, Find the principal.
3. Find the principal that amounts to R5724.5 at 7% per year, in 2 years.
4. Compound Interest of R516.17 at 12% per year, for 4 years, Find the principal.
5. Find the principal that amounts to R2885.79 at 13% per year, in 3 years.
6. Find the compound interest and amount for a principal of R200 at 7% per year, for 2 years
7. Find the principal that amounts to R357.3 at 6% per year, in 3 years.
8. Compound Interest of R179.76 at 14% per year, for 2 years, Find the principal.

Answers:

- 1 $A = R238.2$, $CI = R38.2$
- 2 $P = R600$
- 3 $P = R5000$
- 4 $P = R900$
- 5 $P = R2000$
- 6 $A = R228.98$, $CI = R28.98$
- 7 $P = R300$
- 8 $P = R600$

EXERCISE SET 2

1. A house which cost R75 000 ten years ago was recently sold for R245 000. Calculate the annual rate of appreciation. (Round off your answer correct to 1 decimal digit) (5)
2. If the R75 000 had been invested ten years ago at 10% p.a. compounded half yearly, how much would this investment be worth today? (3)
3. Priscilla invested R1 800 for 6 years. At the end of this term it was worth R3 277. Use a suitable formula to calculate the rate of interest of the investment (round answer off to 1 decimal digit) if interest is compounded annually. (7)
4. A sum of R40 000 is invested for four years at 6.5% interest per annum, compounded annually. The compounded amount is re invested for another four years at 8% interest per annum, compounded half yearly. What is the total interest (correct to the nearest rand) earned over 8 years? (9)

5. Bongani and Barry each have R1500. Barry invests his money for 5 years at 10% p.a. compounded annually. Bongani invests his R1500 for 5 years at 10% p.a. compounded half-yearly. What is the difference in their accrued amounts at the end of the 5-year period? (6)
6. The data collected in a certain town shows that 7% of the population is infected with HIV. It is estimated that half the population will be infected in 15 years' time. (Assume that the growth rate is constant.) If there are 122 500 people in this town, determine, to the nearest percentage:
 - 6.1 The number of people infected with HIV. (1)
 - 6.2 The number of people that will be infected with HIV in 15 years time. (1)
 - 6.3 The rate per annum at which HIV is being spread. (4)
7. The annual expenses of a municipality increase by 9% each year. If the annual expenses in the year 2001 are R 375 million, calculate the projected expenses in the year 2005. (6)
8. What sum of money must be invested now, in order to receive R 16 000 after 4 years, if interest is calculated at 18 % per annum, compounded annually? Round off your answer to the nearest rand. (5)
- 9.1 When Sarah was born, her grandfather deposited R 5 000 at 12,5% per annum compounded annually. The investment remained in the bank until Sarah's 18th birthday. Calculate how much money had accumulated. (3)
- 9.2 If Sarah's grandfather had wanted R 50 000 to be available on her 18th birthday for her university education, and he had deposited the R5 000 in a different bank, determine the rate of interest (compounded annually) that he would have needed to obtain. Give your answer to two decimal digits. (4)
10. A school received a donation of R100 000. The Governing Body is able to invest this at 15% p.a. compounded monthly. Calculate how much interest will be collected by the end of the first year of investment. (4)
11. A new house costs R350 000. It is reported that the price of residential properties increases at a rate of 12% per year, compounded monthly. What will the same house cost in 12 years time? Round your answer off to the nearest thousand.) (6)