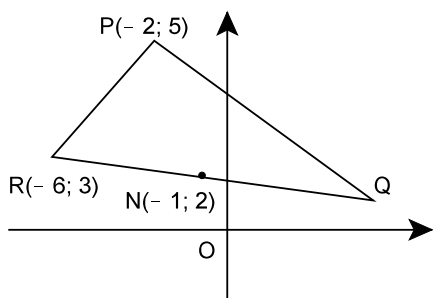


LECTURE 7: ANALYTIC GEOMETRY

APPLICATIONS ON CIRCLES AND LOCI

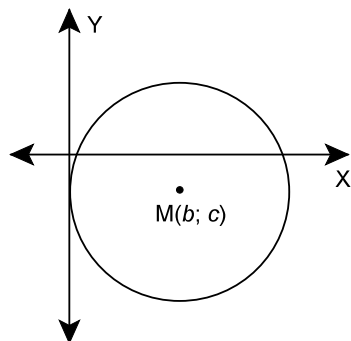
1.1 A(-2 ; 4), B (-6 ; 2) and C (3 ; p) are points in the Cartesian plane. Calculate the value of p if $AB \perp AC$ (5)

1.2 In the diagram alongside, P(- 2; 5), R(- 6; 3) and Q are the vertices of ΔPQR . N(- 1; 2) is the midpoint of RQ.



- 1.2.1 Calculate the size $\angle R$, rounded off to TWO decimal digits. (6)
- 1.2.2 Determine the equation of PN. (4)
- 1.2.3 Determine the coordinates of L, the midpoint of PQ. (5)
- 1.2.4 Determine the coordinates of A, the point of intersection of the medians of ΔPRQ . (6)

2.1 In the diagram alongside, a circle with centre M (b; c) touches the y-axis at point P, where b and c are integers. Point Q (3; 1) lies on the circle. M lies on the straight line $2x + y = 4$ [P and Q are not shown on the diagram.]



- 2.1.1 Write down the coordinates of P in terms of b and c. (2)
- 2.1.2 Determine the equation of the circle in terms of b and c. (2)
- 2.1.3 Hence, determine the numerical values of b

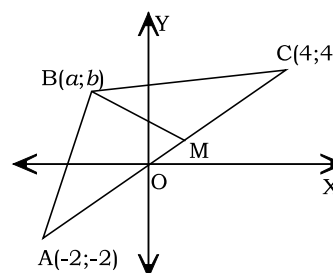
and c. (8)

2.1.4 Hence, determine the equation of the tangent to the circle at Q. (2)

2.2 A circle with equation $x^2 + y^2 + 2x - 6y - 6 = 0$ is given. P is a locus point in the Cartesian plane such that the length of the tangent line drawn from P to the circle is equal to the length of the radius of the circle.

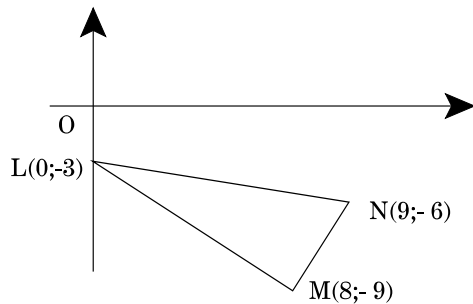
- 2.2.1 Determine:
 - 2.2.1.1 the length of the radius of the circle. (3)
 - 2.2.1.2 the equation of the locus of P. (4)
- 2.2.2 Describe fully the locus obtained in QUESTION 2.2.1.2. (3)

3. In the accompanying x diagram, A(- 2; - 2), B(a; b) and C(4; 4) are the vertices of ΔABC in a Cartesian plane. B is a point in the second quadrant, as indicated, and $AB = BC$. M is the midpoint of AC and the area of ΔABC is equal to 12 square units.



- 3.1 Determine the coordinates of M. (2)
- 3.2 Determine the length of AC and write your answer in the simplest surd form, if necessary. (3)
- 3.3 Now show in detail that the length of BM is equal to $\frac{4}{\sqrt{2}}$ units. (4)
- 3.4 If it is further given that $a + b = 2$, calculate the coordinates of B. (8)

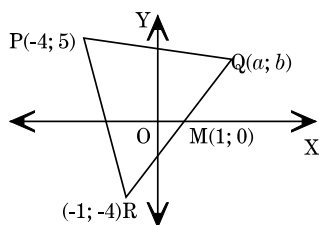
4. $L(0 ; -3)$, $M(8 ; -9)$ and $N(9 ; -6)$ are the vertices of a triangle in a Cartesian plane. $P(x ; y)$ is a point outside $\triangle LMN$ such that the equation of LP is $x - ky - 21 = 0$ and the gradient of PM is -7 .



- 4.1 Show by calculation that $k = 7$. (2)
 4.2 Prove that $LP \perp PM$. (2)
 4.3 Prove that $PLMN$ is a cyclic quadrilateral. (5)
 4.4 Determine the coordinates of the center of the circle through P , L , M and N . (2)
 4.5 Calculate the values of x and y . (7)
 4.6 Calculate the length of PM . (2)
 4.7 Determine the equation of the line parallel to MN and passing through L . (3)
 4.8 Calculate the coordinates of the point where the line in 4.7 cuts the x -axis. (2)

5. Determine the equation(s) of the tangent(s) to the circle $(x - 1)^2 + (y + 2)^2 = 10$ at the point(s) on the circle where $x = -2$. (11)

6. In the diagram, $P(-4 ; 5)$, $Q(a ; b)$ and $R(-1 ; -4)$ are the vertices of a triangle in a Cartesian plane and $M(1 ; 0)$ is the midpoint of QR .



- 6.1 Calculate the values of a and b . (4)
 6.2 Determine the equation of the perpendicular bisector of QR . Give your answer in the form $y = mx + c$. (5)
 6.3 If the perpendicular bisector of PQ has the equation $7x - y + 8 = 0$, and cuts the perpendicular bisector of QR at T , calculate the coordinates of T . (5)
 6.4 Determine the equation of the circumcircle of $\triangle PQR$ and write your answer in the form $ax^2 + bx + cy^2 + dy + e = 0$ (6)
 6.5 Determine the equation of the tangent to the circumcircle of $\triangle PQR$ at point P . (5)