LECTURE 6: APPLICATIONS OF CALCULUS

Thus far we have stressed the meaning of the derivative:

The slope of the tangent to a curve having equation y = f (x) at the point (a, f(a))on the curve is equal to f′(x) that is, the derivative of f (x) evaluated at a.

What can you say about the slope of a curve at its highest and lowest points?

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.....

Since the derivative vanishes (that is it equals 0) at the turning points, we have a powerful way to find the highest and lowest values of a function: we simply make the derivative equal to 0.

8.1 Facing up to word problems in the calculus section.

The majority of learners shy away from word problems. Yet there are always easy marks to score even with problems which look frighteningly unfriendly.

8.1.1 (1) Look for the question that says "find the largest/ smallest/ minimum/ maximum..."
(2) This quantity will be visible in the question as a formula that usually has to be derived.
(3) Write down this quantity.
(4) Differentiate it, using the symbols given in the problem. Numbers other than the variable, like π, for example must be regarded as constants.
(5) Say, "for maximum/minimum, as the case may be, the derivative equals zero" and write the equation, viz. the derivative set equal to zero.
(6) Solve this equation for the unknown.
(7) If the actual maximum or minimum value is asked for, substitute these values into the *original* function (not the derivative) and choose the larger (for maximum) and the smaller (for minimum).

8.1.2 In problems involving rates of change, first identify the symbol for the quantity that is changing. Calculate its derivative *with respect to* t where t is time. Proceed as in 8.1.1.

8.1.3 In all word problems, you will have to read the problem carefully and understand exactly what the question requires. Identify the quantity /quantities that need to be determined and give them symbol names (like x, or r for radius, and so on). Use the information given to set up equations in these unknowns and solve.

PROBLEMS

1. Assume that the average mass of a baby in its first 30 days of life is given by this equation:

$$m(t) = \frac{t^3}{45000} - \frac{t^2}{4500} + 3{,}2 \ ; 0 \le t \le 30.$$

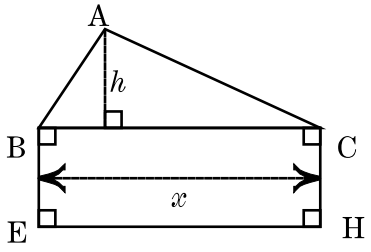
where t is the time in days and m the mass in kilogram.

1.1 What is the baby's mass at birth according to this scale? (2)

1.2 When does the baby's mass reach a minimum? (6)

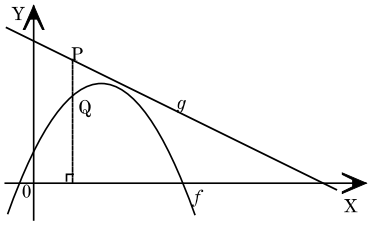
1.3 After how many days would the baby's mass be the same as its mass at birth? (6)

2. In the figure, ΔABC has a base of length x metres. The base and the perpendicular height add up to 10 metres. The triangle is mounted on a rectangle BCHE which has a perimeter of 32 metres. If the composite figure ABEHC has a maximum area, determine the corresponding value of x. (8)



3. Refer to the figure. Sketches have been drawn of :

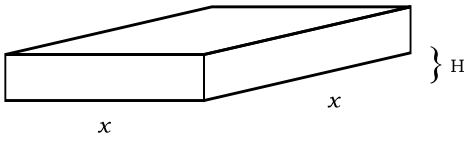
$f(x) = -2(x-1)^2 + 8$



$g(x) = -2x + 11$

Calculate the *minimum* of a line segment PQ, parallel to the y-axis, which can be drawn from g to f. (6)

4. A big open-top rectangular container with a square base has to be made of metal plate. The volume of the container should be 108 m^3. Let the length of the sides of the base be x meters, and the height be h metres.



4.1 Deduce that the surface area, M, of the container will be: $M = \frac{432}{x} + x^2$ (4)

4.2 Determine the value of x and the height of the container that will yield a minimum surface area. (6)

5. Prove that $f(x) = \frac{10}{x^2}$ has no maximum or minimum values. (4)

6. A COKE can has a volume of 340 ml or cm^3. What should the dimensions be, if COKE wants to build the most economical can? (10)