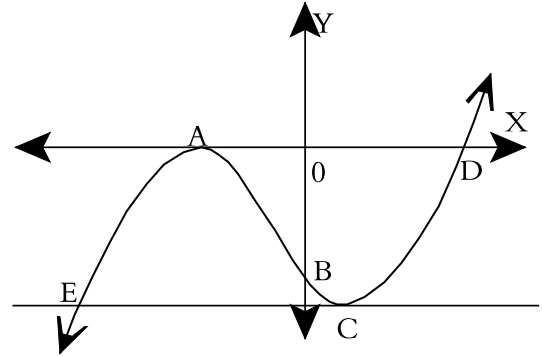


LECTURE 5: CURVE SKETCHING

- 1.1 Sketch the graph of f where $f(x) = x^3 - 6x^2 + 9x$, showing clearly all intercepts on the axes and the co-ordinates of the turning points. (12)
- 1.2 **Use your graph** to answer the following questions:
- (1) For which values of k will the equation $x^3 - 6x^2 + 9x - k = 0$ have three different real roots? (2)
- (2) For which values of x is $f'(x) < 0$? (2)

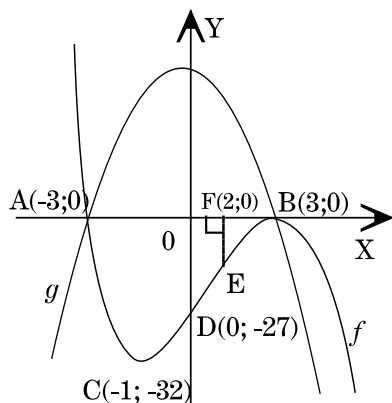
2. The sketch represents the following function $g(x) = x^3 - 3x^2 - 9x - 5$.



The graph turns at A and C. A, B and D are the intercepts with the coordinate axes.

- 2.1 Determine the length of AD. (8)
- 2.2 If CE is a tangent to the curve of g at C, determine the x coordinate of E. (10)
- 2.3 What is the gradient of g at point B? (2)

3. Refer to the figure.
 3.1 Use the graphs given to determine the values of x for each of the following conditions.



- (1) $f(x)$ is a strictly increasing function. (2)
 (2) $f(x).g(x) < 0$ (3)
 (3) $f'(x)$ and $g(x)$ are both negative. (3)

4. Given: $f(x) = -x^3 + 3x^2 + 9x - 27$ where f is the graph drawn in 7.1. Determine:
 4.1 the equation of the tangent to f at the point E (OF \perp FE). (5)
 4.2 the coordinates of the points where the tangent in (b)(1) meets f again. (5)

5. The graph of the curve $f(x) = ax^3 + bx^2$ has a stationary point at $P(1; -\frac{1}{6})$.
 5.1 Show that $a + b = -\frac{1}{6}$ and then prove that $a = \frac{1}{8}$ and $b = -\frac{1}{2}$. (7)
 5.2 Determine the x -intercepts of f . (3)
 5.3 Sketch the graph of f , showing clearly the intercepts on the axes and the turning points. (7)