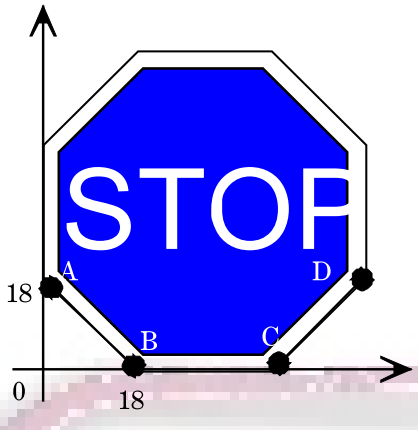
# LECTURE 4: ANALYTIC GEOMETRY II: APPLICATIONS AND CIRCLES

1. A stop sign is placed in a Cartesian plane

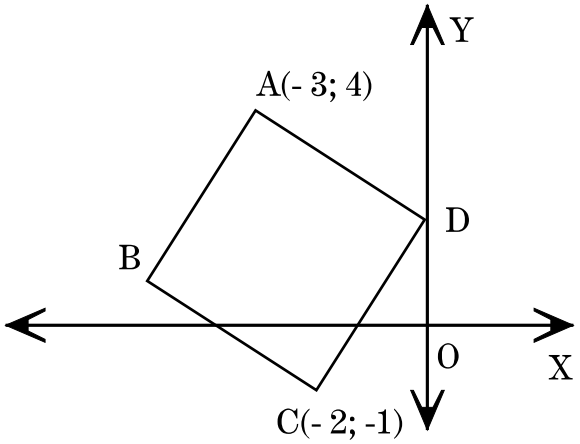



as shown. A stop sign is regular octagon (ie. All eight sides are equal in length).

Find, leaving your answer in surd form where necessary:

1.1 the equation of AB. (3)
1.2 the coordinates of C. (4)
1.3 the equation of CD. (4)

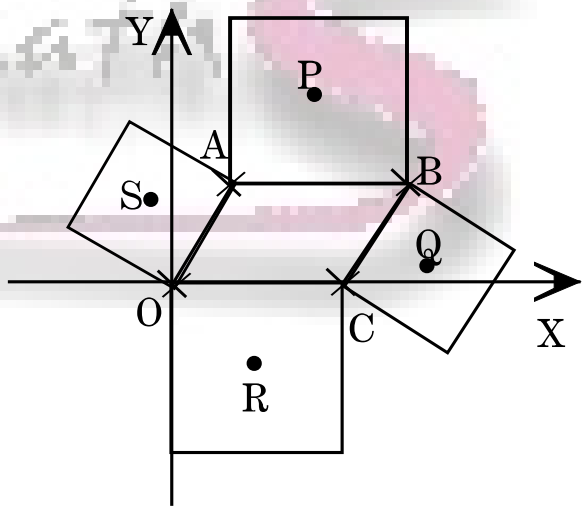
2. The points A(− 3; 4), B, C(− 2; − 1) and D are the vertices of a square ABCD in a Cartesian plane. D is a point on the Y axis.




The diagonals AC and BD intersect at M.

8.1 Show that the length of the diagonal AC is $\sqrt{26}$ units. (2)
2.2 Determine the coordinates of M. (2)
2.3 Calculate the length of any side of the square. (3)
2.4 Determine the coordinates of D. (6)
2.5 Calculate the coordinates of B. (2)

3. Refer to the figure. Parallelogram OABC has coordinates O (0 ; 0), A (1 ;3), B(5 ; 3) and C (4; 0) respectively. Squares are drawn on the sides of OABC as shown. The centers of these squares P, Q, R and S are




joined to form a quadrilateral PQRS. The coordinates of S and Q are (−1 ; 2) and (6 ; 1) respectively.

3.1 Write down the coordinates of P and R. (3)
3.2 Prove that PQRS is a square. (6)

# CIRCLES AND TANGENTS

**Equation of a circle:** with centre the origin (0; 0) is $x^2 + y^2 = r^2$ and with centre $(h; k)$ the equation is

$$(x - h)^2 + (y - k)^2 = r^2.$$

(a) **Locus:** The locus is the set of all points that obeys certain given conditions.

For example: *What is the locus of all points that stay exactly 5 cm from the origin?* The locus is a circle of radius 5 cm, that is $x^2 + y^2 = 25$.

*What is the locus of all points that are 2 cm from the y–axis?* The locus is two straight lines on either side of the $y$ axis and parallel to the $y$ axis. That is they have equations $x = \pm 2$.

4. $x^2 + 4x + y^2 + 2y - 8 = 0$ is the equation of a circle with centre M in a Cartesian plane.

4.1 Prove that the circle passes through the point N(1; – 3). (2)

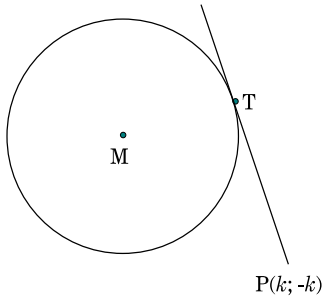
4.2 Determine the equation of PN, the tangent to the circle at N. (8)

4.3 Calculate θ, the angle of inclination of the tangent, rounded off to ONE decimal digit. (2)

4.4 Determine the coordinates of the point where the tangent in QUESTION 2.1.2 intersects the X axis. (2)

4.5 Calculate the coordinates of the point(s) where the circle with centre M cuts the Y axis. (5)

5. Refer to the figure below. M is the centre of a circle with equation $(x - 1)^2 + (y - 2)^2 = 2$. P is any point outside the circle with coordinates




$(k; -k)$. A tangent PT is drawn from P touching the circle at T.

5.1 Show that $PT^2 = 2k^2 + 2k + 3$. (5)

5.2 Determine the length (rounded off to one decimal digit) of the shortest possible tangent that can be drawn from P to the circle. (5)