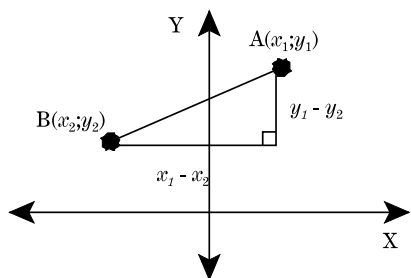


### LECTURE 3: ANALYTIC GEOMETRY

1. **The distance formula:** The distance  $d$  between two points  $A(x_1; y_1)$  and  $B(x_2; y_2)$  is given by  $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ . Note that this is just the theorem of Pythagoras.

2. **Gradient of AB** is given by  $m(AB) = \frac{y_1 - y_2}{x_1 - x_2}$ .

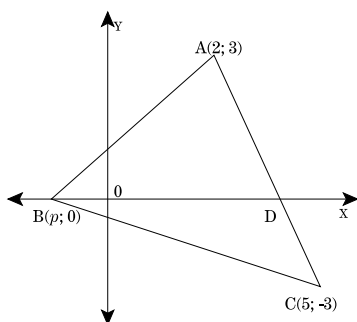


Note that if lines are parallel, they have equal gradients. If lines are perpendicular, the product of their gradients = - 1.

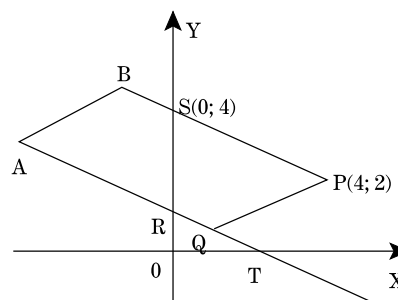
3. The angle  $\theta$  a line makes with the positive  $x$  axis is obtained from  $\tan \theta = m$  the gradient.
- 1.4 **Midpoint Formula:** The midpoint of AB is given by  $\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ .

More generally the if point  $P(x; y)$  divides AB in the ratio  $m: n$ , then  $(x; y) = \left(\frac{mx_2 + nx_1}{m+n}; \frac{my_2 + ny_1}{m+n}\right)$ . Observe that when  $m = n$  we get precisely the midpoint formula above.

1. In the diagram alongside, points  $A(2; 3)$ ,  $B(p; 0)$  and  $C(5; -3)$  are the vertices of  $\triangle ABC$  in a Cartesian plane.
- 1.1 Calculate the coordinates of  $O$ .
- 1.2 Calculate the value of  $p$  if  $BC = AC$  and  $p < 5$ .
- 1.3 Determine the angle of inclination of straight line AC, rounded off to ONE decimal digit. (3)
- 1.4 If  $p = -1$ , calculate the size of  $\angle A$ , rounded off to ONE decimal digit. (3)

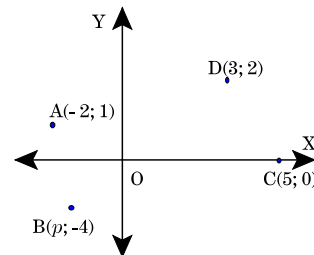


2. The straight line  $x + 3y = 3$  cuts the  $x$ -axis at point A and the  $y$  axis at point B.
- 2.1 If C is the point  $(-5; 4)$  find the coordinates of D, the midpoint of AC. (3)
- 2.2 Determine the length of median BD of  $\triangle ABC$ , leaving your answer in simplified surd form. (3)
3. In the figure below, ABPQ is a quadrilateral. P is the point  $(4; 2)$  and S is the point  $(0; 4)$ . AQ cuts the  $y$  axis at point R.  $PS \parallel QR$  and  $PQ = \sqrt{5}$  units. The gradient of PQ is  $\frac{1}{2}$  and  $x_Q \leq 4$ .



- Determine
- 3.1 the coordinates of point Q. (8)
- 3.2 the equation of straight line QR. (3)
- 3.3 the length of AB, without using the distance formula, given that the equation of AB is  $2y - x = 12$ . (2)
- 3.4 the size of  $\angle PRQ$  rounded off to one decimal digit. (4)

4.  $A(-2; 1)$ ,  $B(p; -4)$ ,  $C(5; 0)$  and  $D(3; 2)$  are the vertices of trapezium ABCD in a Cartesian plane with  $AB \parallel DC$ .
- 4.1 Show that  $p = 3$ . (4)
- 4.2 Calculate  $AB:CD$  in simplest form. (5)
- 4.3 If  $N(x; y)$  is on AB and NBCD is a parallelogram, determine the coordinates of N. (4)
- 4.4 Determine the equation of the line passing through B and D. (1)
- 4.5 What is the angle of inclination of line BD? (1)
- 4.6 Calculate the area of parallelogram NBCD. (4)
- 4.7  $R(-2; q)$ , A and C are collinear. Calculate the value of  $q$ . (4)



5.  $A(1; 3)$ ,  $B(3; -1)$  and  $C(-1; y)$  are the vertices of isosceles  $\triangle ABC$ , with  $AB = BC$ . Calculate the values of  $y$ . (5)
6.  $A(t; t+2)$ ,  $B(t^2; 3t)$ ,  $C(2t; t-3)$  and  $O(0; 0)$  are 4 distinct points.  $t \neq 0$  or 1. Calculate the value of  $t$  if:
- 6.1  $AB \parallel OC$  (6)
- 6.2  $OB \perp AC$  (leave your answer in surd form). (6)