LECTURE 2: DIFFERENTIAL CALCULUS

In lecture 1 we studied the following:

- the notion of a limit (Remember Zeno!)
- the meaning of average gradient
- the meaning of instantaneous gradient
- derivatives (the Newton Quotient)
- introduction to differentiation from first principles

PROBLEMS FROM PAST EXAMINATIONS

1. Find the average rate of change of the function $f(x) = x^2 - 3x + 5$ between $x = 2$ and $x = 7$. (4)

2. Consider $f(x) = -\frac{4}{x}$.
Calculate the average rate of change (average gradient) of the function in the interval $x = 3{,}99$ to $x = 4{,}01$. Approximate the answer correct to six decimal digits. (4)

3. If $f(x) = 3x^2 - 2$, determine $f'(x)$ from first principles. (5)

4. $f(x) = 3x^2 - 2$ (use 3 above)

4.1 Determine the gradient of the tangent to the curve of $f(x)$ at the point where $x = 2$. (3)

4.2 At which point (give the coordinates) on the curve of $f(x) = 3x^2 - 2$ will the gradient be 24? (3)

5.1 Given the function $f(x) = 5x^2 - 3x$, determine $\frac{f(x+h) - f(x)}{h}$ and hence find $f'(x)$. (5)

5.2 Find the equation of the tangent to $f(x)$ at $x = 4$. (4)

6.1 If $y = 3x - x^2$ determine $\frac{dy}{dx}$ from first principles. (5)

6.2 Find the equation of a tangent to $f(x)$ which is parallel to the line $y = -2x + 3$. (5)

6.3 Find the equation of the tangent to $f(x)$ which has an inclination of 60°. (5)

7. In the formula $s = 44t - 6t^2$, s is the distance in metres that is traveled by a motor car in t seconds after the brakes are applied.

7.1 Show that the speed at the time when the brakes were first applied was 158,4 kilometres per hour (4)

7.2 Solve for t if $\frac{ds}{dt} = 0$. (2)

7.3 How many meters did the car travel until it stops? (2)

7.4 If there was a stationary vehicle 60 metres in front of the car at the time of braking, would the car have collided with the stationary vehicle? (1)

RULES FOR DIFFERENTIATION

1. Differentiate with respect to x:

1.1 $3 - x^3$ (1)

1.2 $\sqrt{x} + 3x - 4x^{-2}$ (3)

1.3 $\frac{2x^2 - 7x + 3}{x - 3}$ (3)

2. If $xy = 2 - x\sqrt{x}$ determine $\frac{dy}{dx}$. Give the answer with positive exponents. (4)

3.1 Find: $D_x(x^3 - 2x^2)(2 - x)$ (3)

3.2 $z = \frac{1}{y} + \sqrt[3]{y}$ and $y = \frac{3}{x^2}$ find: $\frac{dz}{dx}$ (4)

4.1 Given: $y = \left(x^2 - \sqrt{x}\right)^2$. Find $\frac{dy}{dx}$. (4)

4.2 Given: $8x^3 - 2xy + y - 1 = 0$, $\left(x \neq \frac{1}{2}\right)$ Evaluate $\frac{dy}{dx}$ when $x = 8$. (5)

4.3 $f(x) = ax^3 + bx^2 + cx - 5$. The gradient at any point
$(x\,;\,f(x))$ is given by $(6x^2 - 24)$. Find the values of a, b and c. (4)

5.1 If $f(t) = \frac{(t-3)^2}{t}$ where $t \neq 0$, determine $f'(t)$. (4)

5.2 A particle moves in a straight line so that after t seconds the distance S (in meters) it traveled is given by
$S = t^3 + 2t^2 + 15t$.
Find the initial velocity of the particle. (4)

5.3 The line $y = 2x - 3$ is a tangent to the curve of $y = x^2 + ax + b$ at the point (2 ; 7). Calculate the values of a and b. (7)

5.4 If $y = \frac{3}{x^2}$ and $z = \frac{1}{y} - y$, determine:

5.4.1 $\frac{dy}{dx}$ (2) 5.4.2 $\frac{dz}{dx}$ (4)

6.1 Determine $D_x(x^2 - a)(b - x)$ if a and b are constants. (3)

6.2 Determine $f'(x)$ in each of the following:

6.2.1 $f(x) = (\sqrt{x} + 2x^2)$ (3)

6.2.2 $f(x) = \frac{(1-x)^2}{x^2}$ (5)