LECTURE 1: DIFFERENTIAL CALCULUS

Please copy notes on these from the slide presentation
- Notion of a limit (The Paradox of Zeno)
- The derivative
- Differentiation Rules

LIMITS

Discuss the following: $\lim_{p\to 3}\left(p^3+1\right)$ and $\lim_{x\to 3}\frac{x^2-9}{x-3}$

[Note the behaviour from the Microsoft Excel spreadsheet].

PROBLEMS

1. Determine: $\lim_{x\to 2}\frac{x^2+x-6}{4-x^2}$ (4)

2. Evaluate: $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ (4)

3. Find: $\lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x+2}-\frac{1}{2}\right)$. (4)

4. Determine : $\lim_{n\to 0}\frac{2(x-n)^2-2x^2}{n}$ (4)

5. Determine: $\lim_{x\to 2}\frac{x\left(x^2-x-2\right)}{-2+x}$ (3)

6. Evaluate: $\lim_{t\to 0}\frac{\sqrt{t+4}-2}{t}$ (4)

7. Calculate: $\lim_{x\to -1}\frac{\sqrt{x^2+8}-3}{x+1}$ (5)

DIFFERENTIATION FROM FIRST PRINCIPLES

copy notes on "average rate of change" and "instantaneous rate of change" from slide presentation.

Newton Quotient:

$$f'(x)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

Notations:

$\frac{dy}{dx}$ *(Leibniz)*
$f'(x)$
$D_x[y]$

Speed (velocity) = rate of change of displacement: $v = s'(t)$
Acceleration = rate of change of velocity: $a = v'(t)$.

1. Find the average rate of change of the function $f(x) = x^2 - 3x + 5$ between $x = 2$ and $x = 7$. (4)

2. Consider $f(x) = -\frac{4}{x}$.
Calculate the average rate of change (average gradient) of the function in the interval $x = 3{,}99$ to $x = 4{,}01$. Approximate the answer correct to six decimal digits. (4)

3. If $f(x) = 3x^2 - 2$, determine $f'(x)$ from first principles. (5)

4. $f(x) = 3x^2 - 2$ (use 3 above)
4.1 Determine the gradient of the tangent to the curve of $f(x)$ at the point where $x = 2$. (3)
4.2 At which point (give the coordinates) on the curve of $f(x) = 3x^2 - 2$ will the gradient be 24? (3)

5.1 Given the function $f(x) = 5x^2 - 3x$, determine $\frac{f(x+h)-f(x)}{h}$ and hence find $f'(x)$. (5)
5.2 Find the equation of the tangent to $f(x)$ at $x = 4$. (4)

6.1 If $y = 3x - x^2$ determine $\frac{dy}{dx}$ from first principles. (5)
6.2 Find the equation of a tangent to $f(x)$ which is parallel to the line $y = -2x + 3$. (5)
6.3 Find the equation of the tangent to $f(x)$ which has an inclination of 60°. (5)

7. In the formula $s = 44t - 6t^2$, s is the distance in metres that is traveled by a motor car in t seconds after the brakes are applied.
7.1 Show that the speed at the time when the brakes were first applied was 158,4 kilometres per hour. (4)
7.2 Solve for t if $\frac{ds}{dt} = 0$. (2)
7.3 How many meters did the car travel until it stops? (2)
7.4 If there was a stationary vehicle 60 metres in front of the car at the time of braking, would the car have collided with the stationary vehicle? (1)