## LECTURE 4: NUMBER PATTERNS

A ***sequence*** is a set of numbers that obey a certain rule or pattern.
Each number in the sequence is called a ***term***. The terms are normally denoted by $T_1$; $T_2$; $T_3$; etc for the first, second, third terms and so on.

**EXAMPLES**

Write down the next three numbers in the following well known sequences.

1. 2; 4; 6; 8; 10; 12; ...
2. 3; 6; 9; 12; ...
3. 1; 4; 7; 10; 13; 16; ...
4. 1; 4; 9; 16; ...
5. 1; 2; 4; 8; 16; ...
6. $\frac{1}{2};\frac{1}{3};\frac{1}{4};\frac{1}{5};...$
7. 2; 3; 5; 7; 11; 13; ...
8. 1; 1; 2; 3; 5; 8; 13; ...
9. 2; 5; 7; 12; 19; ...
10. 2; 6; 12; 20; 30; ...

## ARITHMETIC SEQUENCE

Examine the following sequences below. Can you predict the next 3 terms?

2; 5; 8; 11; 14; ...
5; 11; 17; 23; ...
14; 10; 6; 2; – 2; ...

All of these are Arithmetic sequences. A sequence of numbers such that there is a common difference between successive terms is called an Arithmetic sequence.

**EXAMPLE 1**: 2; 5; 8; 11; 14; ... Find the 1000$^{th}$ term.

**SOLUTION:** Let us write the terms in the following way, keeping in mind that we add 3 to get the next number each time.

| Term no. | Term | Disguise | Short form |
|---|---|---|---|
| $T_1$ | 2 | 2 | 2 |
| $T_2$ | 5 | 2 + 3 | 2 + 3 |
| $T_3$ | 8 | 2 + 3 + 3 | 2 + (2).3 |
| $T_4$ | 11 | 2 + 3 + 3 + 3 | 2 + (3).3 |
| $T_5$ | 14 | 2 + 3 + 3 + 3 + 3 | 2 + (4).3 |
| $T_6$ | 17 | | 2 + (5).3 |

Write $T_7$; $T_8$ and $T_9$ in the same short form as above and find their values.
$T_7$ = 2 + (.....).3 = 20
$T_8$ = 2 + (.....).3 = 23
$T_9$ = 2 + (......).3 = 26

**MAKE A CONJECTURE:** This is fantastic! We can now use this pattern to predict any term. What is the 20$^{th}$ term?
$T_{20}$ = 2 + (19).3 = 59
So the 1000$^{th}$ term would be
$T_{1000}$ = 2 + (.........). 3 = ...............

Say we wish to find term number $n$, where $n$ stands for any number. The formula is
$T_n$ = 2 + (........). 3 = .........................

Repeat the above procedure, for the second and third examples given.

Conjecture the 1000$^{th}$ terms in each case.
Devise a formula to find term $n$.

In fact we can conjecture a more general formula for all arithmetic sequences with first term $a$ and common difference $d$.
In the above example $a = 2$ is the first term, $d = 3$ is the common difference, thus we can conjecture the formula:

$$T_n = a + (n - 1)d$$

as the general term of any arithmetic sequence with first term $a$ and difference $d$.

**EXERCISE**
(*All the sequences in the exercises are arithmetic unless otherwise stated*)

1. In each arithmetic sequence below, find
   a. the next three terms
   b. the $100^{th}$ term
   c. term 801
   d. a formula for term $n$.

1.1 − 2; 3; 8; 13; 18; ...
1.2 12; 19; 26; 33; ...
1.3 22; 19; 16; 13; ...
1.4 101; 81; 61; 41; ...
1.5 3; 12; 21; 30; ...

2. Find the $40^{th}$ term in the arithmetic sequence 17; 14; 11; 8; ...
3. Decide whether 305 can be a term of the sequence 4; 11; 18; 25; ...
4. Which term of the sequence 2; 8; 14; 20; ... is 338?
5. Which term of the sequence − 21; − 17; − 13; − 9; ... is 127?
6. How many negative terms does the sequence − 53; − 47; − 43; ... have?
7. In the sequence 2; 6; 10; ... which term is 82? Is 124 a term of this sequence?
8. How many terms are there in each of these arithmetic sequences?

8.1 7; 9; 11; ... ; 171
8.2 3; 7; 11; ... ; 699
8.3 301; 298; 295; ... ; − 29

## GEOMETRIC SEQUENCE

INVESTIGATE the behaviour of the following sequences in the same way as we have done for Arithmetic Sequences. That is, find the next few terms, try to predict a general formula for any term.

1. 1; 2; 4; 8; 16; ...

2. − 1; 3; − 9; 27; .....

3. 64; 32; 16; 8; ...

These sequences are called GEOMETRIC SEQUENCES.
How do they differ from arithmetic sequence?

In each example above, can you write down the $10^{th}$ term and the $100^{th}$ term. Note a calculator may be used here.

## OTHER SEQUENCES

Study the following sequences carefully. Can you predict the next term? Can you postulate a formula to find any term?

1. 5; 7; 11; 17; 25; ....

2. 3; 5; 10; 18; 29; 43; ...

3. 80; 72; 76; 70; 60; 56; .....

What is the $100^{th}$ term in each of these sequences?

How does the formula for the general term compare to the one for arithmetic sequences.

## PROBLEM SOLVING

1. The value of
   $100^2 - 99^2 + 98^2 - 97^2 + ... - 3^2 + 2^2 - 1^2$ is

   (A) 5 050 (B) 4 950 (C) 5 000
   (D) 25 000 (E) 10 100

2. The $2000^{th}$ letter in the sequence

   *ABCDEDCBAABCDEDCBAABCDEDCB AABC . . .*
   is

   (A)A (B)B (C) C
   (D) D (E) E

3. In the sequence

   $. . . ,k, m, n, p, 0, 1, 1, 2, 3,5, 8, . . .$

   each term is the sum of the two terms on its left. The value of $k$ is

   (A) − 2 (B) 3 (C) − 3
   (D) 2 (E) − 1

17. Consider the pattern shown below:

| | | | | |
|---|---|---|---|---|
| row 1: | 1 | | | |
| row 2: | 3 | 5 | | |
| row 3: | 7 | 9 | 11 | |
| row 4: | 13 | 15 | 17 | 19 |

etc.

The number at the end of row 80 is

(A) 6479 (B) 6319 (C) 6481
(D) 6379 (E) 6531