TURBOMATHS SUPPORT CLASSES

University of KwaZulu Natal

MATHEMATICS Grade 11 HG Lecture 3

EXPONENTS

SURDS

A surd is a number whose square root cannot be found exactly. For example $\sqrt{2}$; $\sqrt{3}$; $\sqrt{17}$.

Surds have similar laws to exponents.

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ And $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$. More generally we can have $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$.

RATIONALISING DENOMINATORS OF SURDS

A fraction like $\frac{1}{\sqrt{2}}$ is a surd and the denominator is irrational. To make the denominator rational we multiply both numerator and denominator by $\sqrt{2}$ so we get $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$. Now the denominator is rational.

Try this out for $\frac{2}{\sqrt{3}}$ and $\frac{5}{\sqrt{5}}$.

SIMPLIFICATION

1. Simplify, without using a calculator:
   $\frac{5^{a-2}.2^{a+2}}{10^a - 10^{a-1}.2}$. (5)

2. Simplify fully: $\frac{4^x + 3.2^{2x-1}}{2^{-x}.2^{3x+2}}$. (5)

3. Without using a calculator, verify that:
   $\sqrt{12} - \sqrt{147} + 3^{1,5} = -2\sqrt{3}$ (4)

4. Evaluate *without using a calculator*:
   $\left(2 - \frac{\sqrt{7}}{2}\right)^{\frac{1}{2}} . \left(2 + \frac{\sqrt{7}}{2}\right)^{\frac{1}{2}}$ (4)

5. Simplify: $\frac{\sqrt{75} - 3\sqrt{12}}{x} = \frac{\sqrt{48}}{2}$ (No calculator.) (4)

4. Find the integer that is the closest approximation to $\frac{10^{2001} + 10^{2003}}{10^{2002} + 10^{2002}}$. (4)

EXPONENTIAL EQUATIONS

1. Solve: $(x + 3)^{-\frac{2}{3}} = 4$. (3)

2. Solve for x: $2^{x^2 - 9} = 1$ (3)

3. Solve for x, without using a calculator:

3.1 $3x^{\frac{2}{3}} - 12 = 0$ (3)

3.2 $3^{x^2 - 1} = \frac{27^{-x}}{3}$ (4)

4. Given that $2^{x+1} + 2^x = 3^{y+2} - 3^y$ where x and y are integers, find the values of x and y. (5)

EXPONENTIAL EQUATIONS THAT BECOME QUADRATICS

1. Solve for x **without using a calculator**.
   $2^{x+1} + 7 = 2^{2-x}$ (6)

2. Solve for x: $3^x + 3(3^{-x}) = 4$ (7)

3. Solve: $2.2^x - 8.2^{-x} = 15$ (6)

4. Consider: $y + 6.3^{x-1} = 3^{2x}$ & $\left(\frac{y+5}{4}\right)^{\frac{1}{x}} = 3$ $x \neq 0$.

4.1 Express y in terms of x for each of the simultaneous equations. (3)

4.2 Hence solve for x: (Give your answer(s) rounded off to one decimal digit where necessary). (5)