

TURBOMATHS SUPPORT CLASSES

University of KwaZulu Natal

MATHEMATICS Grade 11 HG Lecture 2

EXPONENTS

Review of Exponents using the websites:

www.purplemath.com

www.math.com

1. Laws of Indices :

1st Law: $a^x \cdot a^y = a^{x+y}$

2nd Law: $\frac{a^x}{a^y} = a^{x-y}$

3rd Law: $(a^x)^y = a^{xy}$

4th Law: $(ab)^x = a^x b^x$

2. The meaning of a^x :

(1) Firstly $a^0 = 1$ for any number a . Justify.

(2) $a^{-x} = \frac{1}{a^x}$. Justify.

RATIONAL EXPONENTS:

What does $2^{\frac{1}{2}}$ mean? Or $5^{\frac{2}{3}}$?

Suppose the value of $2^{\frac{1}{2}} = x$. Now we raise both side to the power 2 and use LAW 3 on the LHS:

$$\left(2^{\frac{1}{2}}\right)^2 = x^2$$
$$2 = x^2$$

This gives us that $x = \sqrt{2}$. In other words $2^{\frac{1}{2}} = \sqrt{2}$.

In the second case, suppose $5^{\frac{2}{3}} = M$. Raise both sides

to power 3: $\left(5^{\frac{2}{3}}\right)^3 = M^3$

$$5^2 = M^3$$

Now we see that we may write $M^3 = \sqrt[3]{5^2}$ by taking the cube roots of both sides. Thus the meaning of

$5^{\frac{2}{3}}$ is $\sqrt[3]{5^2}$.

Example: Simplify $8^{\frac{2}{3}}$

Solution: One approach is as follows: $8^{\frac{2}{3}} = \sqrt[3]{8^2}$

$$= \sqrt[3]{64}$$
$$= 4$$

EXERCISE

1. Write the following using $\sqrt{\quad}$ form:

1.1 $3^{\frac{1}{2}}$ 1.2 $4^{\frac{1}{5}}$ 1.3 $7^{\frac{1}{9}}$

1.4 $4^{\frac{2}{5}}$ 1.5 $7^{\frac{3}{4}}$ 1.6 $25^{\frac{3}{2}}$

2. Convert to exponent form:

2.1 $\sqrt{31}$ 2.2 $\sqrt[3]{24}$ 2.3 $\sqrt[4]{4^5}$

2.4 $\sqrt[4]{64^7}$ 2.5 $\sqrt[3]{5^x}$ 2.6 $\sqrt[9]{x^8}$

3. Find the value of

3.1 $16^{\frac{1}{2}}$ 3.2 $64^{\frac{1}{4}}$ 3.3 $81^{\frac{3}{4}}$

3.4 $4^{1\frac{1}{2}}$ 3.5 $32^{0.5}$

SIMPLIFICATION OF EXPONENTS

1. Convert to Exponential form and simplify:

1.1 $\sqrt[3]{8y^6}$ 1.2 $\sqrt[5]{32c^{10}a^{15}}$

1.3 $\sqrt[4]{81x^8y^8}$ 1.4 $\sqrt[6]{\frac{m^{12}n^{24}}{64}}$

2. Simplify and write in root form:

2.1 $\left(2a^{\frac{3}{2}}\right)\left(4a^{\frac{1}{2}}\right)$ 2.2 $\left(\frac{x^6}{9b^{-4}}\right)^{\frac{1}{2}}$

2.3 $\left(\frac{x^{\frac{2}{3}}}{4y^{-2}}\right)^{-\frac{1}{2}}$ 2.4 $\frac{x^{\frac{2}{3}}y^{\frac{5}{6}}}{x^{-\frac{1}{3}}y^{\frac{1}{2}}}$

3. Write with a single root sign:

3.1 $\sqrt[3]{6} \cdot \sqrt{2}$

3.2 $\sqrt[4]{xy} \sqrt[3]{x^2y}$

3.3 $\sqrt{x^3} \sqrt[3]{x^2}$

3.4 $\frac{\sqrt[4]{(x+y)^2} \cdot \sqrt[3]{(x+y)}}{\sqrt{(x+y)^3}}$

4.1 Simplify completely (without the use of a calculator):

4.1.1 $8^{-\frac{2}{3}}$ (2)

4.1.2 $(\sqrt[3]{a} \cdot \sqrt{b})^6$ (3)

4.1.3 $\frac{2 \times 7^{2a-1} + 7^{2a+1}}{49^a}$ (4)

5.1 Express the following as an exponent of base 2:

$(\sqrt{8})^6$ (3)

5.2 Simplify:

5.2.1 $\frac{25^k \times 5^{k-1}}{5^{3k-1}}$ (4)

3.1 Simplify (without use of a calculator):

3.1.1 $\left(7\frac{1}{9}\right)^{-0.5}$ (4)

3.1.2 $\left(2 - \frac{\sqrt{7}}{2}\right)^{\frac{1}{2}} \left(2 + \frac{\sqrt{7}}{2}\right)^{\frac{1}{2}}$ (4)

3.1.3 $\left[\frac{2x^{-2}}{(2x)^{-2}}\right]^{\frac{1}{3}}$ (5)

2.1 Simplify (without using a calculator):

2.1.1 $\left(\frac{24}{4}\right)^{\frac{1}{3}}$ (3)

2.1.2 $\left(2^{\frac{1}{2}} - 1\right) \left(2^{\frac{1}{2}} + 1\right)$ (3)

2.1.3 $\frac{4^{x+1} 8^{x+1}}{32^x}$ (5)

6. Use the fact that $(\sqrt{x} - \sqrt{y})^2 = x + y - 2\sqrt{xy}$ to find

the square root of $9 - \sqrt{80}$ (4)

14. Given $2^x = 5^y = 10^z$. Show that: $z = \frac{xy}{x+y}$ (4)

16. Without using a calculator determine which of these is the larger: $4\sqrt[3]{2}$ or $3\sqrt[3]{5}$. (3)