

## LECTURE 6: QUADRATIC EQUATIONS

[Candidates that are doing well in this aspect may proceed to the section: For advanced Students]

### STANDARD FORM: $ax^2 + bx + c = 0$

Solve for  $x$ :

1.1  $(x - 2)^2 = 169$

1.2  $(2x + 5)^2 = 225$

1.3  $(4x - 1)^2 = 3$

1.4  $(x^2 - 3)^2 = 36$

### COMPLETING THE SQUARE

Eg. Solve:  $x^2 - 4x + 3 = 0$

Solution:  $x^2 - 4x + \dots - \dots + 3 = 0$

$$(x \quad )^2 = \dots$$

Solve the following quadratic equations by completion of the square:

2.1  $x^2 - 2x - 24 = 0$

2.2  $2x^2 - 3x - 4 = 0$

2.3  $5x^2 + 2x = 1$

2.4  $2x^2 - bx = 8$ , (in terms of  $b$ ),

### THE QUADRATIC FORMULA

Derive the quadratic formula:

The roots of  $ax^2 + bx + c = 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the formula to solve for  $x$ :

3.1  $2x^2 - 5x + 3 = 0$

3.2  $4x^2 + x - 1 = 0$

3.3  $x^2 - x - 1 = 0$

3.4  $x^4 - x^2 - 4 = 0$

3.5  $x^2 + 2x + 7 = 0$

4. Given  $(x - 1)(3x^2 + x - 7) = 0$   
 4.1 Find all solutions of the equation. (6)

4.2 Find all rational solutions. (1)

**EQUATIONS REDUCIBLE TO A QUADRATIC**

Solve

1.1  $3x^2 - 2x + \frac{1}{3x^2 - 2x} = 2.$

1.2  $\frac{x^2 + 4x}{3} + \frac{84}{x^2 + 4x} = 11$

1.3  $(x^2 - 2x)^2 + 24 = 11(x^2 - 2x)$

1.4  $(\sqrt{2x - 5} + 1)(\sqrt{2x - 5} - 3) = -3$

**FOR ADVANCED STUDENTS**

Solve for  $x$ :  $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

- HINTS:** 1. Put  $y = x + \frac{1}{x}$  and so  $y^2 = \dots\dots$   
 2. Divide the given equation by  $x^2$

**QUESTION:** Why do you think this sinister substitution works? Look at the coefficients of the  $x$  terms.

**FOR VERY ADVANCED STUDENTS**

A stick is broken up into 2 parts so that the ratio of the short to the long part is the same as the ratio of the long part to the whole stick. What is this ratio?

*This is a very famous ratio what is it called?*

One more : Find the value of  $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}$