LECTURE 1: NUMBER SYSTEM

Write down the notes on the number system from the PowerPoint slide show.

RATIONAL NUMBERS

We say that $\frac{1}{3}$ has the form $\frac{p}{q}$ where p and q are both integers. Numbers of this type are called *Rational Numbers* with symbol Q.
In set form we write
Q = {all numbers that can be written in the form $\frac{p}{q}$, where p and q are integers and q cannot be 0}.
That is a rational number is an integer divided by an integer. That is $\frac{\text{Integer}}{\text{integer}}$.

Why is q not equal to 0?

Rational numbers may be written in different forms:

- as a ratio eg. $\frac{1}{4}$
- as a decimal eg. 0,25
- as a percentage eg. 25%

All of these are equivalent.

OPERATIONS ON RATIONAL NUMBERS

(Learners to attempt first)

Simplify:

1. $\frac{2}{5}+1\frac{1}{3}$
2. $3\frac{1}{2}-\frac{3}{5}+5\frac{1}{4}$
3. $\left(1\frac{1}{2}-\frac{3}{4}\right)\times\left(2\frac{2}{3}\right)$
4. $\left(4\frac{1}{3}-\frac{1}{2}\right)\div\left(\frac{7}{8}\right)$
5. $\dfrac{2\frac{1}{2}}{3\frac{3}{4}}$
6. True or False? Explain.

6.1 $\dfrac{5}{\frac{1}{2}+\frac{3}{4}}=5\times\left(\frac{2}{1}+\frac{4}{3}\right)$

6.2 $\dfrac{10+12}{5+2}=\dfrac{10}{5}+\dfrac{12}{2}$

6.3 $\dfrac{12+10}{2}=\dfrac{12}{2}+\dfrac{10}{2}$

DECIMALS

Terminating Decimals

When a decimal number has a fixed number of digits after the comma we call it a terminating decimal. These are some examples:

0,15 0,23459 12,357298765

Terminating decimals can always be written as rational numbers.
Example:
Write 0,125 as a rational number.

Write 0,14789 as a rational number.

EXERCISE

1. Covert the following decimals as rational numbers in simplest form:

1.1 0,1	1.2 ! 0,64	1.3 1,45
1.4 ! 3,75	1.5 2,235	1.6 5,66

2. Convert the following rational numbers to decimal form:

2.1 $\frac{4}{5}$	2.2 $\frac{36}{120}$	2.3 $\frac{7}{8}$
2.4 $-\frac{33}{110}$	2.5 $\frac{165}{50}$	2.6 $2\frac{14}{25}$

Non-Terminating (recurring) Decimals

All the examples above were terminating decimals. Now try to write the rational number

$\frac{1}{3}$ as a decimal. We have to use long division.

What do you notice?
We get the answer 1,333333333...
The 3s go on forever. This kind of a decimal is called a *non-terminating* or a *recurring* decimal.

INVESTIGATION

Use a calculator to write down the decimal version of the following rational numbers:

1. $\frac{7}{9} =$
2. $\frac{25}{11} =$
3. $\frac{41}{333} =$

What do you notice in each case? Even though the calculator can give you just 8 or so places can you fill in the next places where the calculator stopped?

OBSERVATION:

You may have noticed that in each case either a digit or a set of digits repeat itself. When a set of digits repeats itself, we call this behaviour *periodic*.
Example 1 has period 1 since only 1 digit 7 repeats itself.
Example 2 has period 2, since two digits (27) repeat themselves.
Example 3 has period 3, since the three digits 123 keep repeating themselves.

Try these on your calculator:
$\frac{13}{17} =$
$\frac{5}{23} =$

Are they recurring? Is there a pattern?
Do not forget your calculator can only give about 8 digits. It may not be telling you the whole story.

Here are the expansions of the above two fractions to 100 decimal places. Study them carefully:

$\frac{13}{17} =$ 0.7647058823529411764705882352941176470588235294117647058823529411764705882352941176470588235294117647

$\frac{5}{23} =$ 0.2173913043478260869565217391304347826086956521739130434782608695652173913043478260869565217391304348

You will observe in each case that the decimals are periodic.
What is the period in each case?

CONJECTURE:

We can make the following conjecture after examining the above behaviour.
All rational numbers have either
- a terminating decimal form (finite number of digits)
- or a periodic decimal form.

CONVERTING RECURRING DECIMALS TO RATIONAL FORM

The instructor will show you how to convert recurring decimals to rational form.

Study these three examples carefully. Without working details, can you convert 0,46464646... to decimal form.
Solution: From what we see above, the pattern when 2 digits recur is to have $\frac{46}{99}$. You can check this with a calculator.

Convert 0,621621621... to rational form.
Solution: 0,621621621... $= \frac{621}{999} = \frac{207}{333} = \frac{69}{111} = \frac{23}{37}$.

EXERCISE

1. Convert the following to rational form showing working details

1.1 0,343434...	1.2 3,2525252....
1.3 4,219219...	1.4 0,7
1.5 4,72	1.6 0,0678

2. Without working, convert the following to rational form, leaving your answer in simplest form:

2.1 0,636363... 2.2 1,8
2.3 6,123 2.4 12,42

IRRATIONAL NUMBERS

Can we write all numbers as fractions? How about the number 5? Of course we may disguise 5 as $\frac{5}{1}$ in which case it is rational. The same applies to ! 12. It can be written as $\frac{-12}{1}$ so it is also rational. So are there numbers than cannot be written as an integer divided by an integer? The answer is yes.

For example $\sqrt{2}$ cannot be written as one integer divided by another;
Here is a decimal form of $\sqrt{2}$ to 100 decimal places:

$\sqrt{2}$ = 1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415 73

(as printed:)
1.41421356237309504880168872420969 8
0785696718753769480731766797379907 3
24784621070388503875343276415 73

Is it periodic, can you find a block of numbers that repeat itself? Can you guess what digits come after the 100th digit 3?
Well here is the value of $\sqrt{2}$ to 1000 decimal places:

$\sqrt{2}$ = 1.4142135623730950488016887242096980785696718753769 48
0731766797379907324784621070388503875343276415727350 1
3846230912297024924836055850737212644121497099935831 4
1322266592750559275579995050115278206057147010955997 1
6059702745345968620147285174186408891986095523292304 8
4308714321450839762603627995251407989687253396546331 8
0882964062061525835239505474575028775996172983557522 0
3375318570113543746034084988471603868999706990048150 3
0544027790316454247823068492936918621580578463111596 6
6871301301561856898723723528850926486124949771542183 3
4204285686060146824720771435854874155657069677653720 2
2648544701585880162075847492265722600208558446652145 8
3988939443709265918003113882464681570826301005948587 0
4003186480342194897278290641045072636881313739855256 1
1732204024509122770022694112757362728049573810896750 4
0183698683684507257993647290607629969413804756548237 2
8997180326802474420629269124859052181004459842150591 1
2024944134172853147810580360337107730918286931471017 1
111683916581726889419758716582152128229518488 47

The same behaviour can be seen in the mysterious number *B*. Sometimes we use $\frac{22}{7}$ as an approximation for *B*. But this is a very poor approximation. To 8 decimal places *B* has the value 3,141592. Check this on a calculator.

One cannot say much by looking at 8 places.
Let us look at *B* to 100 decimal places.

B = 3.14159265358979323846264338327950 2
884197169399375105820974944592307 81
6406286208998628034825342117068

Is it periodic? Do you find any set of numbers repeating themselves?

Consider *B* to 1000 decimal places:

B = 3.1415926535897932384626433832795028841971693993751 05
8209749445923078164062862089986280348253421170679821 4
8086513282306647093844609550582231725359408128481117 4
5028410270193852110555964462294895493038196442881097 5
6659334461284756482337867831652712019091456485669234 6
0348610454326648213393607260249141273724587006606315 5
8817488152092096282925409171536436789259036001133053 0
5488204665213841469519415116094330572703657595919530 9
2186117381932611793105118548074462379962749567351885 7
5272489122793818301194912983367336244065664308602139 4
9463952247371907021798609437027705392171762931767523 8
4674818467669405132000568127145263560827785771342757 7
8960917363717872146844090122495343014654958537105079 2
2796892589235420199561121290219608640344181598136297 7
4771309960518707211349999998372978049951059731732816 0
9631859502445945534690830264252230825334468503526193 1
1881710100031378387528865875332083814206171776691473 0
3598253490428755468731159562863882353787593751957781 8
577805321712268066130019278766111959092164201 99

Can you detect any pattern? Frankly there seems to be no real pattern, however the great Indian mathematical genius Srinivasa Ramanujan, is reported to have been able to memorise the value of *B* to several decimal places!

In fact, when an unending number written in decimal form, has digits that do not repeat themselves, such a number is **irrational**. You cannot convert it to the form $\frac{p}{q}$ where p and q are both rational. As an example, consider 0,101001000100001... The digits do not repeat themselves so the number is irrational even though there is a nice pattern that the digits follow.

CONCLUSIONS

Square roots of numbers that cannot be found exactly are irrational.
The number *B* is irrational.
Non-recurring infinite decimals are irrational.
NB. The are a lot more numbers that are irrational but these are the most common at your level.

EXERCISE

1. Classify each of the following numbers using the symbols N (Natural numbers) , N_0 (Whole numbers); Z (Integers); Q (Rational numbers), Q! (Irrational numbers), R (Real numbers), C (Unreal numbers).

1.1 47
1.2 ! 5
1.3 $\sqrt{3}$
1.4 1,212112111211112...
1.5 $\sqrt{36}$
1.6 *B*
1.7 $\frac{22}{7}$
1.8 *B*r^2 if $r = 7$
1.9 $\sqrt{1+\frac{9}{16}}$
1.10 0,474747...

2. If $x = -2 \pm \sqrt{9-4a}$ and a 0 {0; 2; 3}, for which values of a will x be
2.1 real
2.2 rational
2.3 integral (3)